Master Lesson Plan

For

Factorisation

Board	Standard	Subject	Chapter	Language	Reference Link	Creation date
CBSE	STD VIII	Mathematics	Factorisation	English	Factorisation	2019-11-02 21:24:48

Factorisation

1. MS_Objectives_Factorisation

Notes to teacher:

This asset lays down the proposed plan for transacting this chapter. It states the asset objectives of the MLP. This asset is for teacher's reference and need not be taught to students.

Students will be able to

- identify factors of numbers and algebraic expressions
- describe factorisation of algebraic expressions into prime/irreducible factors
- explain factorisation by the method of common factors
- explain factorisation by the method of regrouping the terms
- apply the method of regrouping the terms for factorisation
- appreciate the importance of good character in life
- explain factorisation using identities
- apply the method of factorisation using identities
- become interested in factorisation in Cryptography
- analyze factorisation of a trinomial with leading coefficient 1
- explain factorisation of a quadratic polynomial with positive constant term
- explain factorisation of a quadratic polynomial with negative constant term
- apply different methods of factorisation
- analyze factorisation of trinomials with leading coefficient different from 1
- determine factors of algebraic expressions
- judge that factorisation of polynomials is used by Physicists, Scientists, Astronomers
- identify that factorisation of polynomials are used extensively in our day to day lives
- apply the method of division of polynomials by monomials, polynomials

Time to teach	Asset Type	Theme	SubTheme
3 Minutes	Main Script	Factorisation	

2. IQ_What is Factorisation?

What is a factor of a number?

Expected Answer:

Factor is an exact divisor.

E.g. When 29 is divided by 3; we will get 9 as the quotient and 2 as the remainder.

$29 = 9 \times 3 + 2$. Here 3 does not divide 29

When the remainder is 0, the divisor is called the factor of the number

E.g. When 27 is divided by 3, the quotient is 9 and the remainder is 0

Thus, $27 = 9 \times 3 + 0 = 9 \times 3$

Thus 3, 9 are the factors of the number 27

2) What is a factor of an algebraic expression?

Expected Answer:

When we multiply two or more algebraic expressions, we get another algebraic expression

E.g. when we multiply, x + 1 and x + 2, we get the product,

$(x + 1)(x + 2) = x^2 + 3x + 2$

x + 1 and x + 2 are factors of $x^2 + 3x + 2$

3) What is factorisation?

Expected Answer:

Factorisation of a number/algebraic expression is a common mathematical process of finding the factors or finding what to multiply together to get the number/algebraic expression

Factorisation of algebraic expression is like "splitting" the expression into a product of simpler expressions

The factors of an algebraic expression may be numbers or algebraic expressions.

4) a) What is a prime number or prime algebraic expression?

b) What is unique factorisation theorem?

Expected answer:

a) Prime number:

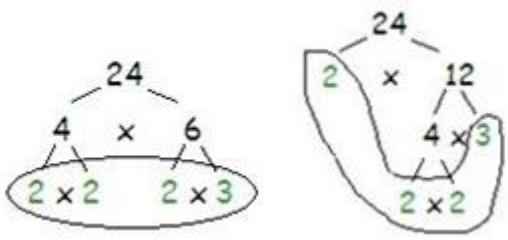
A positive integer greater than 1 that has only two factors (i.e., itself and 1) is termed as prime number

E.g. 2, 3, 5, 7 are some primes

In algebraic expressions, we use the word 'irreducible' in place of 'prime'. We say that an algebraic expression is irreducible if it has only two factors, I and itself

x + 1, x – 3 etc. are some examples of irreducible expressions

b) The unique factorisation theorem states every positive integer larger than 1 can be expressed uniquely as the product of its prime factors except for the order in which the prime factors are written
In other words, we can split any number/algebraic expression into prime/irreducible factors and the splitting is unique



E.g.
We can write, 24 as 4 × 6 or 2 × 12

Now 24 = 4 × 6 = 2 × 2 × 2 × 3

Also 24 = 2 × 12 = 2 × 4 × 3 = 2 × 2 × 2 × 3

Thus 24 has a unique representation 2 × 2 × 2 × 3 as product of primes

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Inquisitive Questions	Factorisation	Factorisation, Factorisation

3. MS_ Introduction to Factorisation

There are three Types of Factorization of an algebraic expression

- Factorization by removing Common Factors
- Factorization by Grouping
- Factorization by using Identities

Before using these methods, we should know common factors and highest common factors of numbers/algebraic expressions

The Highest Common Factor (HCF)

Consider the numbers 18 and 24

The number 3 is a factor of both these numbers .The number 3 is called common factor of 18, 24

Let us list all positive common factors of 18, 24

18 = 1 × 18 = 2 × 9 = 3 × 6

24 = 1 × 24 = 2 × 12 = 3 × 8 = 4 × 6

The list of factors of 18, {1, 2, 3, 6, 9, 18}

The list of factors of 24, {1, 2, 3, 4, 6, 8, 12, 24}

The common factors of 18 and 24 are the factors in both sets {1, 2, 3, 6}

We can see that 2, 3 and 6 are the common factors of 18, 24 other than 1.

The number 6 is the highest among these factors and is called the ‘Highest common Factor'.

Factors of algebraic expressions

The factors of an algebraic expression may be numbers or algebraic expressions.

For example, in the algebraic expression $6x^2y + 7x$ the term $6x^2y$ has been formed by the factors 2, 3, x, x and y, i.e., $6x^2y = 2 \times 3 \times x \times x \times y$.

Observe that the factors 2, 3, x, x and y of $6x^2y$ cannot further be expressed as a product of factors.

We may say that 2, 3, x, x and y are irreducible factors of $6x^2y$.

Note $2\times3 \times (x^2y)$ is not product of irreducible factors of $6x^2y$, since the factor x^2y can be further expressed as a product of x and y, i.e., $x^2y = x \times x \times y$.

Next consider the expression $10\,x\,(y + 7)$.

It can be written as a product of factors, 2, 5, x and (y + 7).

$10\,x\,(y + 7) = 2 \times 5 \times x \times (y + 7)$.

The factors 2, 5, x and (y +7) are irreducible factors of $10\,x\,(y + 7)$.

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factorisation	Factorisation, Factorisation

4. MS_The method of common factors

Consider the factorization of 4x + 6.

1) Write $4x = 2 \times 2 \times x$

$6 = 2 \times 3$

2) The common factor in the two expressions is 2.

3) The highest common factor is also 2.

4) Dividing the expression 4x + 6 by 2.

$4x + 6 = 2\,(2x + 3)$

Therefore, factorization of $4x + 6 = 2\,(2x + 3)$.

$$\begin{aligned} 4x + 6 &= 4 \times x + 6 \\ &= \boxed{2} \times 2 \times x + \boxed{2} \times 3 \\ &= \boxed{2} \times 2x + \boxed{2} \times 3 \\ &= 2(2x+3) \end{aligned}$$

Note: We can check the answer by multiplying using distributive law.

2(2x + 3) = 4x + 6

Example 2: Consider the expression $3x^2 + 15xy$

1) Write $3x^2 = 3 \times x \times x$

$15xy = 3 \times 5 \times x \times y$

2) The common factors in the two expressions are 3 and x

3) The highest common factor is 3x

4) Dividing the expression $3x^2 + 15xy$ by 3x

$3x^2 + 15xy = 3x(x + 5y)$

Therefore, factorization of $3x^2 + 15xy = 3x(x + 5y)$

Example 3: Factorize 3y + 15

Both 3y and 15 have a common factor of 3

3y is 3 × y

15 is 3 × 5

So, we can factorize the whole expression into:

3y + 15 = 3 × y + 3 × 5

= 3 (y + 5) (using the reverse of distributive property)

Thus, 3y + 15 = 3(y + 5)

So, 3y +15 has been factorized into 3 and y+5

When you take out a common factor make sure that it is the highest common factor, including any variables.

E.g. Factorize $3y^2 + 15y$

3 and 15 have a common factor of 3

So we could have:

$3y^2 + 15y = 3(y^2 + 5y)$

But we can do better!

$3y^2$ and 15y also share the variable y

Together that makes 3y:

$3y^2$ is $3y \times y$ and 15y is $3y \times 5$

So we can factor the whole expression into:

$3y^2 + 15y = 3y \times y + 3y \times 5 = 3y(y + 5)$

Thus, factorization by removing the common factors involves the following simple steps

Step 1) Find the HCF of all the terms of the expression.

E.g. For the expression, $x^2 - 3x$ the HCF of x^2 and 3x is x.

Step 2) Write each term as a product of factors with HCF as one of the factors. In $x^2 - 3x$, $x^2 = x \times x$ and $3x = 3 \times x$.

Step 3) Take out the HCF. Here, we use the reverse of distributive property i.e. ab - ac = a(b - c).

$x^2 - 3x = x(x - 3)$

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factorisation	Factorising by common factors, Factorising by common factors

5. MS_Factorization by regrouping the terms

Consider the factorization of the expression $x^2 + 2x + 7x + 14$

Step 1: Finding a common factor of the expression

Is there a common factor for this expression other than 1?

No.

But we can express it as the sum of two binomial terms and obtain their common factors separately.

Thus $x^2 + 2x + 7x + 14$

$= (x^2 + 2x) + (7x + 14)$

Step 2: Finding a common factor of $x^2 + 2x$

What is the common factor of $x^2 + 2x$?

x is the common factor.

Step 3: Factorise $x^2 + 2x$

$x^2 + 2x = x (x + 2)$

Step 4: Finding a common factor of $7x + 14$

What is the common factor of $7x + 14$?

7 is the common factor.

Step 5: Factorise $7x +14$

$7x + 14 = 7(x + 2)$

Step 6: Write an expression equivalent to $x^2 + 2x + 7x + 14$ which uses the factorization in steps 3 and 5 above.

$x^2 + 2x + 7x + 14 = (x^2 + 2x) + (7x + 14)$

$= x(x + 2) + 7 (x + 2)$

Step 7: Find the common binomial factor in $x(x + 2) + 7(x + 2)$.

It's $x+2$

Step 8: Now use the distributive property to write an expression equivalent to

$x (x + 2) + 7 (x + 2)$

$x (x + 2) + 7 (x + 2) = (x + 2) (x + 7)$

Thus, we have factorized $x^2 + 2x + 7x + 14$ by grouping the terms. In this process, we find a common factor in two of the four terms. Then we obtain same common factor in the other two terms, in such a way that distributive property can be used still one more time.

Note that in the expression, $x^2 + 2x + 7x + 14$, we can also group the terms as $(x^2 + 7x) + (2x + 14)$ $= x(x + 7) + 2(x + 7)$

$= (x + 7)(x + 2)$

Thus, grouping can be done in different ways.

We follow the following steps in the process of **Factorization by grouping**.

Step 1: Arrange the terms of the given expression in suitable groups such that each group has a common factor

Step 2: Factorize each group

Step 3: Take out the factor which is common to each group

In the following algebraic expression, there are six terms. We divide the expression into 3 groups of two terms each and then factorize.

$y^3 - 3y^2 + 2y - 6 - xy + 3x$

$(y^3 - 3y^2) + (2y - 6) - (xy - 3x)$

$= y^2 (y-3) + 2(y-3) - x(y-3)$

$= (y - 3)(y^2 + 2 - x)$

Note: We do not always know how to select pairs of terms for this kind of factorization. Often some rearrangement of terms is necessary.

e.g.

1) $3x^2 - 7x + 6x - 14$

$= 3x^2 + 6x - 7x - 14$

$=3x (x + 2) - 7 (x + 2)$

$= (x + 2) (3x - 7)$

2) $z^3 - z^2 - 9z + 9$

Try factorizing the first two and second two separately.

$= (z^3 - z^2) - (9z - 9)$

$z^2(z - 1) - 9(z - 1) = (z - 1) (z^2 - 9)$

$z^3 - z^2 - 9z + 9 = (z - 1) (z^2 - 9)$

$z^2 - 9$ is a difference of squares.

$(z^2 - 9) = (z - 3) (z + 3)$

So, $z^3 - z^2 - 9z + 9 = (z - 1) (z - 3) (z + 3)$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factorisation	Factorising by regrouping terms, Factorising by regrouping terms

6. QA_Factorize by regrouping the terms

1) $a^2-3a+5a-15$ 2) $5t^2 + 2t - 10t - 4$

3) $4a^2 - 10a + 2a - 5$ 4) $6y^2 + 4y - 9y - 6$

5) $t^2 + 7t + 4t + 28$ 6) $3c^2 - 5c + 15c - 25$

7) $3x^2 +15x + 2x + 10$ 8) $16x^2 +20x - 12x - 15$

Solutions:

1) $a^2 – 3a + 5a – 15 = (a^2 – 3a) + (5a – 15)$

$= a (a – 3) + 5 (a - 3) = (a – 3)(a + 5)$

2) $5t^2 + 2t - 10t – 4$

Rearranging the terms,

$5t^2 + 2t - 10t – 4 = (5t^2 – 10t) + (2t - 4)$

$= 5t (t - 2) + 2 (t – 2) = (t – 2)(5t + 2)$

3) $4a^2 - 10a + 2a - 5$

Rearranging the terms,

$4a^2 - 10a + 2a - 5 = (4a^2 + 2a) (- 10a - 5)$

$= 2a (2a + 1) – 5(2a + 1) = (2a + 1) (2a - 5)$

4) $6y^2 + 4y - 9y – 6$

Rearranging the terms,

$6y^2 + 4y - 9y – 6 = (6y^2 – 9y) + (4y - 6)$

$= 3y (2y – 3) + 2 (2y – 3) = (2y – 3) (3y + 2)$

5) $t^2 + 7t + 4t + 28$

$t^2 + 7t + 4t + 28 = (t^2 + 7t) + (4t + 28)$

$= t (t + 7) + 4 (t + 7) = (t + 7) (t + 4)$

6) $3c^2 - 5c + 15c – 25$

By Rearranging the terms,

$3c^2 - 5c + 15c – 25 = (3c^2 + 15c) + (- 5c – 25)$

$= 3c (c + 5) - 5 (c + 5) = (c + 5) (3c – 5)$

7) $3x^2 + 15x + 2x + 10$

$3x^2 + 15x + 2x + 10 = (3x^2 + 15x) + (2x + 10)$

$= 3x(x + 5) + 2(x + 5) = (x + 5)(3x + 2)$

8) $16x^2 + 20x - 12x - 15$

By Rearranging the terms,

$16x^2 + 20x - 12x - 15 = (16x^2 - 12x) + (20x - 15)$

$= 4x(4x - 3) + 5(4x - 3) = (4x - 3)(4x + 5)$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Assessments	Factorisation	Factorising by regrouping terms, Factorising by regrouping terms

7. VC_Regrouping our Value System

"Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping."

Taking Common Factor by regrouping is one method of Factorisation. For example, a large society is a blend of groups of people with various collective

properties. It is a polynomial of religion, culture, age, income levels, education levels, etc.

The factors here that are common to everyone in that group--all are children of God, all have the need to love and be loved; all want peace in lives.

Children! Please remember how important it is to extend our love to everyone, even if they seem outwardly to be very different from us. If we restrict our love only to our immediate family and friends it comes with attachment and is unwise – if we have love for all beings it is uplifting.

Similarly, when we consider a group of students, the common factors are:

S—Sincerity

T—Tenacity

U—Unity

D—Discipline

E—Energetic

N—Neatness

T—Truthfulness

S—Steadfastness

And what emerges as a common factor here turns out to be “Good character.” Children!

Develop good character as your distinct achievement.

“Ability may take you to the top. But it takes good character to keep you there” John Wooden

Time to teach	Value Type	Value Sub Type	Value Attribute
5 Minutes	Right Action	Good behavior	Open

8. MS_Factorization by using Identities

In factorization, we have to find factors which produce the given expression when multiplied. It is like trying to find out what ingredients are added into a food to make it delicious. It is not easy sometimes

So, it’s worth remembering the following identities as they can make factorization easier

$a^2 - b^2$	=	$(a + b)(a - b)$
$a^2 + 2ab + b^2$	=	$(a + b)^2$
$a^2 - 2ab + b^2$	=	$(a – b)^2$

Factorization using identity difference of squares:

Example: Factorize $9x^2 – 25$

Here, we cannot see any common factor but we can see it as the "difference of squares".

Since $9x^2$ is $(3x)^2$, and 25 is $(5)^2$,

We have:

$9x^2 - 25 = (3x)^2 - (5)^2$

And that can be factorized by the difference of squares formula:

$a^2 - b^2 = (a+b)(a-b)$ where a is 3x, and b is 5.

So, $9x^2–25 = (3x)^2 - (5)^2 = (3x+5)(3x-5)$

Factorization of a square trinomial expression:

We know that the square of a binomial is the sum of the square of the first term, twice the product of the terms and the square of the second term.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

The algebraic expressions in the right hand side of the above formulae are called square trinomials.

We can use the above formulae to factorize square trinomials

E.g. Factorize

1. $x^2 + 14x + 49 = x^2 + 2 \times x \times 7 + 7^2 = (x + 7)^2$

2. $2x^2 – 24x + 72 = 2(x^2 – 12x + 36)$

$= 2(x^2 – 2 \times x \times 6 + 6^2)$

$= 2(x - 6)^2$

How do you distinguish between an ordinary trinomial and a square trinomial?

In a square trinomial, the first term is a square, the third term is a square and the middle term = 2 × square root of the first term × square root of the third term

E.g. $x^2 + 6x + 4$ is not a perfect square. Why?

The first term is x^2, the third term is 2^2 but middle term 6x is not equal to

$2 \times x \times 2$

Later we will study the methods of factorization of trinomials of the type

$x^2 + mx + n$ which are not perfect squares.

When trying to factorise using identities, follow these steps:

- Recognize the suitable identity for the given expression
- Rewrite the expression in the form of the identity
- Write the expression as a product of irreducible factors using the identity

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factorisation	Factorising by using identities, Factorising by using identities

9. QA_Factorize algebraic expressions using Identities

Factorise the following algebraic expressions using identities

I) a) $4 - 16x^2$ b) $100z^4 - 49z^2$ c) $144z^{16} - 1$ d) $81x^6 - 121$ e) $24t^6 - 54$

Solutions:

I) a) $4 - 16x^2 = 2^2 - (4x)^2 = (2 + 4x)(2 - 4x)$

b) $100z^4 - 49z^2 = z^2(100z^2 - 49) = z^2 [(10z)^2 - 7^2] = z^2(10z + 7)(10z - 7)$

c) $144z^{16} - 1 = (12z^8)^2 - 1^2 = (12z^8 + 1)(12z^8-1)$

d) $81x^6 - 121 = (9x^3)^2 - 11^2 = (9x^3 + 11)(9x^3 - 11)$

e) $24t^6 - 54 = 6(4t^6 - 9) = 6[(2t^3)^2 - 3^2] = 6(2t^3 + 3)(2t^3 - 3)$

II) a) $9a^2 + 42a + 49$ b) $49 + 14ab + a^2b^2$ c) $16t^2 - 4t + \frac{1}{4}$ d) $2x^6 + 8x^3 + 8$

e) $x^2 + x + \frac{1}{4}$ f) $1 - 8x + 16x^2$ g) $25x^4 + 30x^2 + 9$ h) $4x^2 - 4x + 1$

Solutions:

II) a) $9a^2 + 42a + 49 = (3a)^2 + 2 \times 3a \times 7 + 7^2 = (3a + 7)^2$

b) $49 + 14ab + a^2b^2 = 7^2 + 2 \times 7 \times ab + (ab)^2 = (7 + ab)^2$

c) $16t^2 - 4t + \frac{1}{4} = (4t)^2 - 2 \times 4t \times \frac{1}{2} + (\frac{1}{2})^2 = (4t - \frac{1}{2})^2$

d) $2x^6 + 8x^3 + 8 = 2(x^6 + 4x^3 + 4) = 2[(x^3)^2 + 2 \times x^3 \times 2 + 2^2] = 2(x^3 + 2)^2$

e) $x^2 + x + \frac{1}{4} = x^2 + 2 \times x \times \frac{1}{2} + (\frac{1}{2})^2 = (x + \frac{1}{2})^2$

f) $1 - 8x + 16x^2 = 1^2 - 2 \times 1 \times 4x + (4x)^2 = (1 - 4x)^2$

g) $25x^4 + 30x^2 + 9 = (5x^2)^2 + 2 \times 5x^2 \times 3 + 3^2 = (5x^2 + 3)^2$

h) $4x^2 - 4x + 1 = (2x)^2 - 2 \times 2x \times 1 + 1^2 = (2x - 1)^2$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Assessments	Factorisation	Factorising by using identities, Factorising by using identities

10. IA_Cryptography and Factorization

What is Cryptography?

In the word crypt means 'hidden' and graphy means 'writing'. Thus, 'Cryptography is the science of hiding the information in order to conceal it from unauthorized access. It converts data into a format that is unreadable for an unauthorized user. The unreadable format is called cipher text and only those who know the secret key can decrypt or decode the message.

For example, around 100 B.C., Julius Caesar was known to use a form of encryption to convey secret messages to his army generals.

So, earlier Roman method of Cryptography was popularly known as 'Caesar Shift Cipher'

In this method, the letters of a message were shifted by an agreed number (3 was a common choice). The receiver of the message would then shift the letters back by the same number (decrypt) and obtain the original message.

For example, by shifting two letters forward of the message 'come at two p.m.' would take the form 'eqog cv vyq r.o.' For a website to be secure, all the data transmitted between the computers where the data is kept and where it is received must be encrypted.

It is very important in Internet. It is used in electronic transaction to protect data such as account numbers and transaction amount. Also, it is used to protect e-mail messages, credit card information etc.

How is Factorization useful in Cryptography?

Here is one example to show how factorization is used in cryptography.

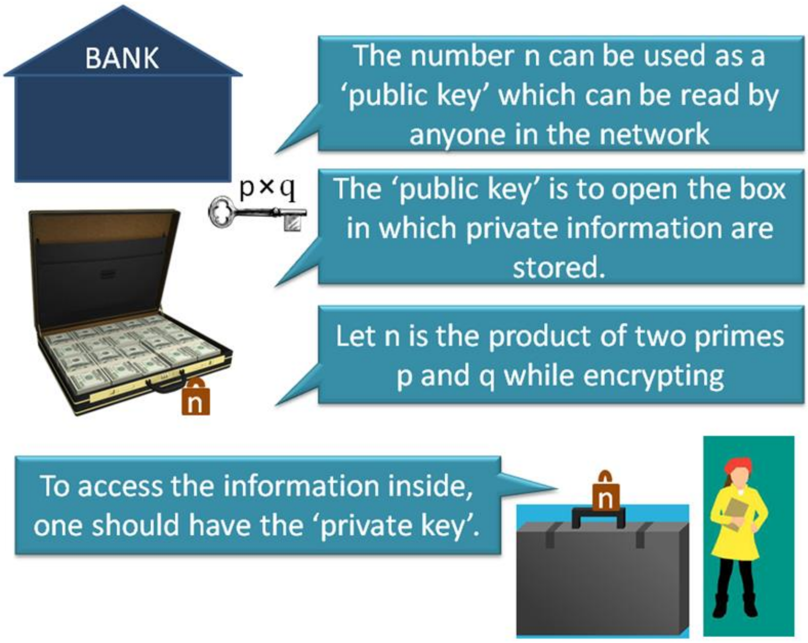
We know that prime numbers play the role of atoms in matters. They are the

fundamental units of every number. E.g. 6 is the product of 2, 3. The number

70 is the product of 10 and 7 in which, 10 is again the product of 2, 5.

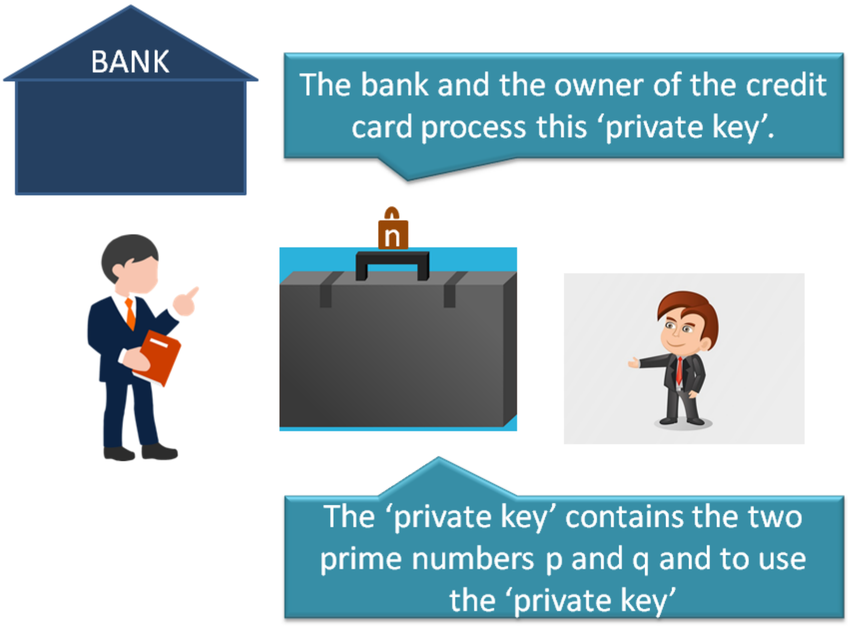
For a computer multiplying two primes, even 100 digit numbers is not difficult, but factorization of the product to get back the primes is difficult and time-consuming. This fact is used to prepare keys as follows.

Let n be the product of two primes p and q. While encrypting, say, for example, credit card details, the number n can be used as a 'public key' which is read by anyone in the network as the name indicates.



The 'public key' is to open the box in which private information are stored. The box can be accessed by the public. To access the information inside, one should have the 'private key'.

The bank and the owner of the credit card process this 'private key'.



The 'private key' contains the two prime numbers p and q and to use the 'private key', the thief (hacker) must factorize n which could take thousands of years by computers if p and q are hundreds of digits long. Factorization is not impossible.

But it's very time consuming. So, it would be impossible for hackers to access the information inside the box even though they open the box using the 'public key'.

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Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Interesting Asides	Factorisation	Factorisation, Factorisation

11.MS_ Factorization of a trinomial with leading coefficient one

Consider a trinomial $x^2 + 5x + 6$

Obviously it is not the difference of squares.

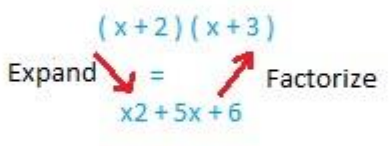
The number 6 is not a perfect square. So, it's not a square trinomial. We can't use $(a + b)^2$ or $(a - b)^2$ formulae to factorize it

To factorize an algebraic expression is to find what to multiply to get it

Let us guess $(x + 2)$, $(x + 3)$ are the factors (As 2, 3 are the factors of 6)

The expansion of $(x + 2)(x + 3)$ is $x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

Thus, factors of $x^2 + 5x + 6$ are $(x + 2)$, $(x + 3)$



The process of Expansion and Factorization are opposite.

We can multiply the factors and obtain the trinomial easily. But every time we cannot guess the factors of a trinomial.

Here is a method of factorization of a trinomial.

Factorization of trinomial $x^2 + mx + n$

Let x + a, x + b be its factors

Then $(x + a)(x + b) = x^2 + mx + n$

$x^2 + ax + bx + ab = x^2 + mx + n$

$x^2 + (a + b)x + ab = x^2 + mx + n$

Equating the coefficients we get, a + b = m and ab = n

Therefore, find two numbers a, b such that their sum is m and product is n. Then,

x + a, x + b are the factors of $x^2 + mx + n$.

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Main Script	Factorisation	Factors of other form, Factors of other form

12.MS_ Factorization of a quadratic polynomial with positive constant term

Definition: The absolute value or magnitude of a number 'a' is denoted by |a| and is defined as

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example:

| 7| = 7 because 7 > 0 and |–7| = – (–7) = 7 as – 7 < 0

Thus we see that |7| = |–7| = 7

Let the trinomial be $x^2 + m x + n$ where n is positive and m is positive or negative.

We find two numbers a, b such that a + b = m and ab = n

If the product of two numbers is positive then the numbers have the same sign.

Therefore use the following steps:

Step1: **Find | m | and two positive factors a, b of n whose sum is | m |**

Step2: What are the signs of the factors?

Take both a, b positive if m is positive and a, b negative if m is negative

Step3: In the algebraic expression $x^2 + mx + n$ replace mx by ax + bx. The expression is $x^2 + ax + bx + ab$.

Step4: Factorize this expression by the method of regrouping the terms.

Example 1: Factorize $x^2 + 13x + 22$

Solution:

Step 1:

m = 13, |m| =13 and n = 22

Let us find the positive values of 'a' and 'b' so that a × b = 22 and a + b = 13

a	b	a + b	Is a +b = lml = 13?
1	22	23	No
2	11	13	Yes

The positive factors of 22 are 2, 11

Step 2: What are the signs of a, b?

m is positive. Both a, b are positive.

Thus a = 2, b = 11

Step 3: Write $x^2 + 13x + 22$ as $x^2 + 2x + 11x + 22$

Step 4:

$x^2 + 2x + 11x + 22 = (x^2 + 2x) + (11x + 22)$

$= x(x + 2) + 11(x + 2) = (x + 2)(x + 11)$

Example 2: Factorize x2 - 21x + 90

Solution:

Step 1:

m = -21, |m| =21 and n = 90

Let us find the positive values of 'a' and 'b' so that a × b = 90 and a + b = 21

a	b	a + b	Is a +b = lml = 21?
1	90	91	NO
2	45	47	NO
3	**30**	**33**	**NO**
5	**18**	**23**	**NO**
6	**15**	**21**	**YES**

The positive factors of 90 are 6, 15

Step 2: What are the signs of a, b?

m is negative. Both a, b are negative

Thus a = - 6, b = - 15

Step 3: Write $x^2 - 21x + 90$ as $x^2 - 6x – 15x + 90$

Step 4:

$x^2 - 6x – 15x + 90 = (x^2 - 6x) - (15x – 90)$

$= x(x - 6) - 15(x - 6) = (x - 6)(x - 15)$

Time to teach	Asset Type	Theme	SubTheme
8 Minutes	Main Script	Factorisation	Factorisation, Factorisation

13.MS_ Factorization of a quadratic polynomial with negative constant term

Let the trinomial be $x^2 + mx + n$ where n is negative and m is positive or negative.

We find two numbers a, b such that a + b = m and ab = n

If the product of two numbers is negative then the numbers have the opposite sign.

Therefore, use the following steps:

Step 1: **Find |m| and two positive factors of n whose difference is |m|.**

Step 2: What are the signs of the factors?

Let the bigger factor take the sign of m and the other factor take the opposite sign.

Step 3: In the algebraic expression $x^2 + mx + n$ replace mx by ax + bx. The expression is $x^2 + ax + bx + ab$.

Step 4: Factorize this expression by the method of regrouping the terms.

Example 1: Factorize $x^2 + 7x – 18$

Step 1:

$m = 7$, $|m| = 7$ and $n = -18$, $|n| = 18$

Let us find the positive values of 'a' and 'b', a >b so that a × b = 18 and a - b = 7

a	b	a - b	Is a - b = ImI = 7?
18	1	17	No
9	2	7	Yes
6	3	3	No

The positive factors of 18 are 9, 2

Step 2: What are the signs of a, b?

The bigger number a takes the sign of m and b takes opposite sign.

m is positive. Therefore a = +9 and b = - 2

Step 3: Write $x^2 + 7x – 18$ as $x^2 + 9x – 2x - 18$

Step 4: $x^2 + 9x - 2x - 18 = (x^2 + 9x) - (2x + 18)$

$= x(x + 9) - 2(x + 9) = (x + 9)(x - 2)$

Example 2: Factorize $x^2 – 5x – 24$

Step 1:

$m = -5$, $|m| = 5$ and $n = -24$, $|n| = 24$

Let us find the positive values of 'a' and 'b', a >b so that a × b = 24 and a - b = 5

a	b	a - b	Is a - b = ImI = 5?
24	1	23	No
12	2	10	No
8	3	5	Yes
6	4	2	No

The positive factors of 24 are 8, 3

Step 2: What are the signs of a, b?

The bigger number a takes the sign of m and b takes opposite sign.

m is negative. Therefore a = - 8 and b = 3

Step 3: Write $x^2 – 5x – 24$ as $x^2 - 8x + 3x - 24$

Step 4: $x^2 - 8x + 3x - 24 = (x^2 - 8x) + (3x - 24)$

$=x(x - 8) + 3(x - 8) = (x - 8) (x + 3)$

Time to teach	Asset Type	Theme	SubTheme
8 Minutes	Main Script	Factorisation	Factorisation, Factorisation

14. QA_Factorize the following

To factorize $x^2 + mx + n$

The following rules are used

If n is positive

1) Find |m|. Find two positive factors of n such that their **SUM** is |m|.

2) Assign **SAME** sign to the factors, i.e. the sign of m.

If n is negative

1) Find |m|. Find two positive factors of n such that their **DIFFERENCE** is |m|.

2) Assign **DIFFERENT** signs to these factors.

Bigger factor will get the sign of m and the other factor will get the opposite sign.

Factorize the following:

1) $x^2 – 2x – 35$ 2) $x^2 + 13x + 36$ 3) $x^2 – 17x + 72$ 4) $x^2 + 18x – 40$

Solutions

1) $x^2 – 2x – 35$

m = - 2, |m| = 2 and n = -35

Step 1: n is negative

So, find two positive factors of 35 such that their difference is |m| = 2.

The list of positive factors is, (35, 1) and (7, 5).

The factors satisfying the condition are 7 and 5.

Step 2: What are the signs of factors?

They have opposite signs as n is negative.

m is negative. The bigger factor 7 has a negative sign and 5 has a positive sign.

a = -7 and b = 5

Step 3: $x^2 – 2x – 35 = x^2 – 7x + 5x – 35 = x(x -7) + 5(x – 7) = (x – 7) (x + 5)$

2) $x^2 + 13x + 36$

m = 13, |m| = 13 and n = 36

Step 1: n is positive.

So, find two positive factors of 36 such that their sum is |m| = 13.

The list of positive factors is, (36, 1), (18, 2), (12, 3), (9, 4) and (6, 6).

The factors satisfying the condition are 9 and 4.

Step 2: What are the signs of factors?

They have same signs as n is positive.

m is positive. Therefore, both factors have positive sign.

a = 9 and b = 4

Step 3: $x^2 + 13x +36 = x^2 + 9x + 4x + 36 = x(x + 9) + 4(x +9) = (x + 9)(x + 4)$

3) $x^2 – 17x + 72$

m = - 17, |m| = 17 and n = 72

Step 1: n is positive

So, find two positive factors of 72 such that their sum is |m| = 17

The list of positive factors is, (72, 1), (36, 2), (24, 3), (18, 4) and (9, 8)

The factors satisfying the condition are 9 and 8

Step 2: What are the signs of factors?

They have same signs as n is positive.

m is negative. Therefore, both factors have negative sign.

a = - 9 and b = - 8

Step 3: $x^2 – 17x + 72= x^2 - 9x - 8x +72 = (x^2 - 9x) – (8x - 72) = x(x – 9) - 8(x – 9)$

$= (x – 9)(x – 8)$

4) $x^2 + 18x – 40$

$m = 18$, $|m| = 18$ and $n = -40$

Step1: n is negative.

So, find two positive factors of 40 such that their difference is $|m| = 18$.

The list of positive factors is, (40, 1), (20, 2), (10, 4), (8, 5).

The factors satisfying the condition are 20 and 2.

Step2: What are the signs of factors?

They have opposite signs as n is negative

m is positive. The bigger factor 20 has a positive sign and 2 has a negative sign

$a = 20$ and $b = -2$

Step 3: $x^2 + 18x - 40 = x^2 + 20x - 2x - 40 = x^2 + 20x - (2x + 40)$

$= x(x + 20) - 2(x + 20) = (x + 20)(x - 2)$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Assessments	Factorisation	Factorisation, Factorisation

15. MS_ Factorization of trinomials with leading coefficient different from one

To factorise trinomials of the type $px^2 + qx + r$

Consider product of the coefficient of x^2 and constant term $p \times r = n$ and

Coefficient of $x = q = m$

Now the following rules are used to factorise $px^2 + qx + r$

If n is positive

1) Find $|m|$

Find two positive factors of n such that their **SUM** is $|m|$

2) Assign **SAME** sign to these factors. I.e. the sign of m

If n is negative

1) Find $|m|$

Find two positive factors of n such that their **DIFFERENCE** is $|m|$

2) Assign **DIFFERENT** signs to these factors.

Bigger factor will get the sign of m and the other factor will get the opposite sign

Examples:

1) $2x^2 + 7x + 6$

Step 1:

The given trinomial is $px^2 + qx + r$ with $p = 2$, $q = 7$ and $r = 6$

Let $n = p \times r = 2 \times 6 = 12$ and $m = q = 7$

Step 1: n is positive

So, find two positive factors of 12 such that their sum is $|m| = 7$

The list of positive factors of 12 is (12, 1), (6, 2) and (4, 3)

The factors satisfying the condition are 4, 3

Step 2: What are the signs of factors?

They have same signs as n is positive

Also, both are positive as m is positive

Step 3: $2x^2 + 7x + 6$

$= 2x^2 + 4x + 3x + 6$

$= (2x^2 + 4x) + (3x + 6)$

$= 2x(x + 2) + 3(x + 2)$

$= (x + 2)(2x + 3)$

2) Factorise $6x^2 - 11x - 7$

Step 1:

The given trinomial is $px^2 + qx + r$

With $p = 6$, $q = -11$ and $r = -7$

Let $n = pr = 6 \times (-7) = -42$ and $m = q = -11$

$m = -11$, $|m| = 11$

Step 2: n is negative.

So, find two positive factors of 42 such that their difference is $|m| = 11$.

The list of positive factors is, (42, 1), (21, 2), (14, 3) and (7, 6).

The factors satisfying the condition are 14 and 3.

Step 3: What are the signs of factors?

They have different signs as n is negative.

m is negative. Therefore the bigger factor 14 is negative and 3 is positive.

$a = -14$ and $b = 3$

Step 4:

$6x^2 - 11x - 7 = 6x^2 - 14x + 3x - 7 = (6x^2 - 14x) + (3x - 7) = 2x(3x - 7) + 1(3x - 7)$

$= (3x - 7)(2x + 1)$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factorisation	Factorisation, Factorisation

16. SA_ Find the factors of the Algebraic Expression

Aim: To determine factors of given algebraic expression

Materials required: Two medium size boxes, placards, one white sheet, scissor and glue.

Type of the activity: Group

Instruction:

Two days before the activity, let the teacher write a list of enough number of algebraic expressions and their factors. (Approximately the number of expressions is at least one third of the class strength).

Avoid expressions of the form $(a + b)^2$ so that each expression has two distinct factors.

Thus, each expression is associated with two factors. For each expression, choose three students and assign one to an expression and to the rest, a factor each. For example, if the expression is $x^2 + 5x + 6$ then its factors are $x + 2$ and $x + 3$.

Now assign $x^2 + 5x + 6$ to student 1, $x + 2$ to student 2 and $x + 3$ to students 3.

Repeat it for all. Ask each student to prepare a placard of size 8 cm x 5 cm and write the expression assigned to each.

Procedure:

Step 1: Write 'algebraic expression' in one paper and 'factors' in another. Paste the papers on the boxes.

Step 2: Ask each student to come and put the cards in the respective boxes.

Step 3: The teacher will divide the class into two groups, approximately one-third of students in group A and the remaining students in group B. The students of group A and B are made to sit separately.

Step 4: Each student of group A should come, and pick a card from the first box, return to his/her place and start factorizing the algebraic expression in that card.

Step 5: On the other side of the class room, each student of group B will pick one card from the second box. Then the students are made to stand in one or two rows holding their cards.

Step 6: The student who completes his/her factorization work will move to the other side, try to find students holding the cards with factors calculated by him/her.

Repeat the activity by interchanging students in group A and group B.

Conclusion: The activity makes the students enjoy factorization.

Image Source:

https://pixabay.com/illustrations/kids-play-child-children-playing-1340525/

Time to teach	Asset Type	Theme	SubTheme
20 Minutes	Suggested Activity	Factorisation	Factors(for natural numbers and algebraic expressions), Factors(for natural numbers and algebraic expressions)

17. IQ_ Is every polynomial factorisable?

1) Is it possible to factorize any given algebraic expression/polynomial?

No. $x^2 + x + 1$ is an example of a polynomial which is not factorisable as the product of two linear factors.

Later, in higher classes you will know about complex numbers and every polynomial is factorisable as product of linear factors with coefficients in the set of complex numbers.

2) Why it is useful to know how to factor polynomials?

The most fundamental reason is to be able to solve polynomial equations.

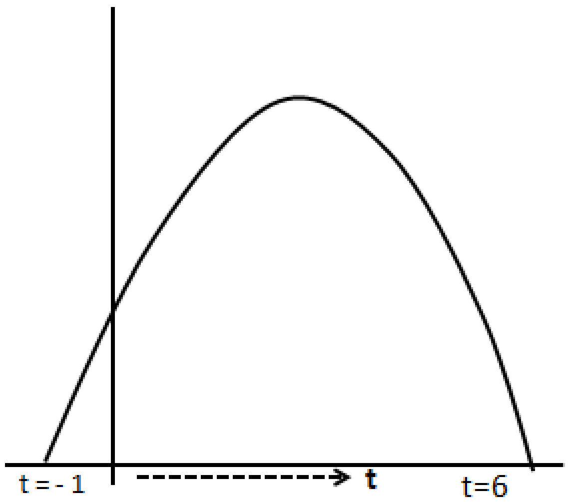
If you have an equation saying that a product of several factors must be equal to zero, the solution is that one or more of the factors must be zero.

For example, suppose you throw a ball into the air and want to find when it hits the ground.

Laws of Physics tell you that the height of the ball, t seconds after you throw it is a quadratic polynomial in t and t = 0 when the ball hits the ground.

A ball is thrown vertically upwards and its height h ft. after any time t secs is given by the formula, $h = -t^2 + 5t + 6$.

To find when the ball hits the ground, you will have to find the value of t, which satisfies the equation $-t^2 + 5t + 6 = 0$.



If you're able to factor the left-hand side, you see that the equation becomes $-(t - 6)(t + 1) = 0$, $t = 6$ or $t = -1$. Ignoring the negative solution, you can see that the ball hits the ground six seconds after you throw it.

Image Source: Original Contribution by saithri@yahoo.co.in

Time to teach	Asset Type	Theme	SubTheme
7 Minutes	Inquisitive Questions	Factorisation	Factorisation, Division of polynomial by a monomial, Factorisation, Division of polynomial by a monomial

18. DD_ Use of Algebraic expressions and factorization

Polynomials can have real world uses. Some carriers require the use of polynomials, and calculation of their roots by factorization.

For Example,

Scientists, physicists, chemists and astronomers need to use polynomials in their job.

Algebraic expressions and factorization help physicists to measure relationships between characteristics like force, mass acceleration etc.

Astronomers get polynomial equations in finding the distances and studying the features of celestial bodies such as stars, planets, moons etc.

Statisticians: They use Mathematical techniques to analyze and interpret data. Their job requires solutions of polynomial equations and factorization when they work in fields like economics, social, political etc.

Engineers: Civil, electrical, mechanical and industrial engineers need mathematical skills and their job requires calculations using polynomial expressions. E.g. Mechanical engineers use polynomials to design engines and machines.

Carpenters, architects and interior decorators use algebraic expressions to calculate the exact quantity of supplies required for a particular area that needs to be landscaped.

In this way we see that Algebraic expressions are used extensively in our day to day lives and so it is important to have knowledge of Algebraic expressions so that we can calculate things easily.

Image source:

Businessman: **https://pixabay.com/illustrations/chart-businessman-growing-african-4105121/**

Cartoon: https://pixabay.com/vectors/sketch-character-funny-man-art-3075369/
Carpenter taking measurements- https://www.pexels.com/photo/person-holding-pencil-1388944/

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Day-to-day Relevance	Factorisation	Factors(for natural numbers and algebraic expressions), Factors(for natural numbers and algebraic expressions)

19.MS_ Division of Algebraic Expressions

Division of Algebraic Expressions

We know that division is the inverse operation of multiplication

Thus, $7 \times 8 = 56$ gives $56 \div 8 = 7$ or $56 \div 7 = 8$

Division of algebraic expressions is explained similarly

For example,

(i) We have $2x \times 3x^2 = 6x^3$

Therefore, $6x^3 \div 2x = 3x^2$ and also, $6x^3 \div 3x^2 = 2x$

(ii) $5x (x + 4) = 5x^2 + 20x$

Therefore, $(5x^2 + 20x) \div 5x = x + 4$

and also $(5x^2 + 20x) \div (x + 4) = 5x$

Division of a monomial by another monomial

For dividing the monomial we need to divide the numerical coefficients of the variables and the variables themselves.

Example 1 : Consider $6x^3 \div 2x$

A shorter way to depict cancellation of common factors is as we do in division of numbers:

$$77 \div 7 = \frac{77}{7} = \frac{7 \text{x} 11}{7} = 11$$

$$\text{Similarly, } 6x^3 \div 2x = \frac{6x^3}{2x} = \frac{\cancel{2} \text{ x } 3 \text{ x } \cancel{x} \text{ x } x \text{ x } x}{\cancel{2} \text{ x } \cancel{x}}$$

$$= 3 \times x \times x = 3x^2$$

Example 2: $-20x^4 \div 10x^2$

Solution:

$-20x^4 = (-2) \times 2 \times 5 \times x \times x \times x \times x$

$10x^2 = 2 \times 5 \times x \times x$

$$\text{Therefore, } (-20x^4) \div 10x^2 = \frac{(-2) \text{ x } \cancel{2} \text{ x } \cancel{5} \text{ x } \cancel{x} \text{ x } \cancel{x} \text{ x } x \text{ x } x}{\cancel{2} \text{ x } \cancel{5} \text{ x } \cancel{x} \text{ x } \cancel{x}} = -2x^2$$

Division of Polynomial by monomial

Step1: When you have to divide a polynomial by a monomial, write it as a fraction with the polynomial in the numerator and monomial in the denominator

E.g. $(4x^3 - 6x^2 + 3x - 9) \div 6x$

Step1: Write it as $\frac{4x^3 - 6x^2 + 3x - 9}{6x}$

Step2: Write the polynomial as the sum or difference of individual items each divided by the monomial in the denominator

I.e. $\frac{4x^3}{6x} - \frac{6x^2}{6x} + \frac{3x}{6x} - \frac{9}{6x}$

Step3: Simplify each term. Use the rules for exponents to simplify the variables in each term

The above quotient after simplification is,

$\frac{2}{3}x^2 - x + \frac{1}{2} - \frac{3}{2x}$

Division of Polynomial by polynomial

Division of a Polynomial with a polynomial can be done in two ways.

First method:

- Write the dividend and divisor as numerator and denominator respectively
- If possible, factorize the numerator
- Cancel the common terms in the numerator and denominator

Example 1:

$(15x^4 - 10x^2 + 3x^2 - 2) \div (3x^2 - 2)$

$$\frac{15x^4 - 10x^2 + 3x^2 - 2}{3x^2 - 2} = \frac{5x^2\left(3x^2 - 2\right) + 1(3x^2 - 2)}{3x^2 - 2} = \frac{\left(3x^2 - 2\right)(5x^2 + 1)}{3x^2 - 2} = 5x^2 + 1$$

Example 2:

$(21m^3 + 7m^2 - 45m - 15) \div (3m + 1)$

$$\frac{21m^3 + 7m^2 - 45m - 15}{3m+1} = \frac{7m^2\,(3m+1) - 15\,(3m+1)}{3m+1} = \frac{(3m+1)(7m^2 - 15)}{3m+1} = (7m^2 - 15)$$

Second method:

Step1: The Dividend and the divisor should be arranged in decreasing order of the degrees of their variables

Step2: Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient

Step3: Multiply the divisor by the first term of the quotient and subtract the product from the dividend. The remainder obtained becomes the next dividend

Step4: Repeat steps 2 and 3 with the new dividend

The steps are repeated till we get a zero or the dividend cannot be further divided and we get a remainder

Example 1

Divide $(x^2 + 8x + 16)$ by $(x+4)$

$$
\begin{array}{r|l}
 & (x+4) \\
\hline
(x+4) & (x^2+8x+16) \\
 & (x^2+4x) \\
\hline
 & 0+4x+16 \\
 & 4x+16 \\
\hline
 & 0
\end{array}
$$

Example 2:

Divide $6a^4 + 15a^3 + 7a^2 + 13a + 56$ by $2a + 3$

$$
\begin{array}{r|l}
 & 3a^3 + 3a^2 - a + 8 \\
\hline
2a+3 & 6a^4 + 15a^3 + 7a^2 + 13a + 56 \\
 & 6a^4 + 9a^3 \\
\hline
 & 6a^3 + 7a^2 \\
 & 6a^3 + 9a^2 \\
\hline
 & -2a^2 + 13a \\
 & -2a^2 - 3a \\
\hline
 & 16a + 56 \\
 & 16a + 24 \\
\hline
 & 32
\end{array}
$$

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factorisation	

20. MS_Summary_Factorisation

- The factors of numbers and algebraic expressions are defined
- The method of factorisation of terms of algebraic expressions into prime/irreducible factors is explained
- Factorisation by the method of common factors and regrouping of terms is explained
- Algebraic expressions are factorized by the method of regrouping of terms
- The importance of good character in life is explained
- The method of factorisation using identities is explained and applied to factorize given algebraic expressions
- The use of factorisation in Cryptography is explained
- Factorisation of a trinomial with leading coefficient 1 is explained

- factorisation of a quadratic polynomials with positive and negative constant terms is explained
- Algebraic expressions are factorised using different methods
- Factorisation of trinomials with leading coefficient different from 1 is explained
- Factors of different types of polynomials are obtained in an activity
- The uses of factorisation of quadratic polynomials in Science, Physics etc are explained by taking one example
- Examples of factorisation of polynomials in our day to day lives are listed
- The method of division of polynomials by monomials, polynomials etc is explained

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Main Script	Factorisation	